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Matter, Mathematics, and God

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Abstract *This paper contrasts materialism and theism in their ability to give a plausible account of mathematics. The ideal, abstract entities of mathematics do not fit readily within a materialist world. Truth, logic, and mathematics require the existence of universal norms. However, materialism has no place for abstract entities or universal norms. How can such ideal norms, inert in themselves, influence our minds? The indispensability of mathematics to physics is a strong argument for realism. Theism posits that God upholds mathematical truths, which reside in the divine mind. Classical mathematics presumes the existence of an Ideal Mathematician—an all-powerful, all-knowing and infinite God. Since theism holds that man is created in God's image and that God can communicate truth to us, humans may be expected to have access to mathematical knowledge. Since God has created the world according to a rational plan, the world may be expected to have a mathematical structure.*

Key words: Mathematics; Materialism; Theism; Realism; Platonism; Universals; Norms; Constructivism

Introduction

The aim of this paper is to examine and contrast the abilities of materialism and theism to account for mathematics. A key question is whether mathematics is a mere human invention or an exploration of an already existent realm.

Historically, most mathematicians believe that mathematical truths (e.g. "$2 + 5 = 7$") exist independently of human minds, being universally and eternally true. Most mathematicians believe they are discovering properties of, say, prime numbers, rather than merely inventing them. This view of mathematics dates back to Pythagoras (ca 569–475 BC) and Plato (427–347 BC). It is often called "Platonism" or mathematical "realism."

Mathematics has led to some profound philosophical speculation. Bertrand Russell, certainly no friend of theism, concluded from his study of the history of Greek philosophy that "Mathematics is . . . the chief source of the belief in eternal and exact truth, as well as in a supersensible intelligible world."[1] This is so, Russell argues, because of the abstract nature of mathematical concepts. For example, geometry deals with exact circles, but no physical object is exactly

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circular. This suggests that exact reasoning applies to ideal, rather than physical, objects. Furthermore, numbers appear to be time-independent. Hence, mathematics seems to deal with an ideal, eternal world of pure thought.

However, where and how do such mathematical entities exist? The existence of eternal, ideal mathematical thoughts seems to require the existence of something actual in which they exist.

The early theistic philosophers Philo (ca 20 BC – 50 AD) and Augustine (354 – 430) placed the ideal world of eternal truths in the mind of God. Augustine argued that mathematics implied the existence of an eternal, necessary, infinite Mind in which all necessary truths exist. He asserted that we all know time-independent truths about logic (e.g. A = A) and mathematics (e.g. 2 + 2 = 4). However, changing, material things cannot cause fixed, eternal truths. Nor can finite human minds, since our thinking does not make them true but rather, it is judged by them. Thus, truth must derive from something non-material that is superior to the human mind. Mathematical truths must depend on a universal and unchanging source that embraces all truth in its unity. Such a Truth, Augustine argued, must exist and is by definition God.[2]

Thus arose the classical Christian view that mathematics exists in the mind of God, that God created the universe according to a rational plan (Proverbs 8), and that man's creation in the image of God (Genesis 1:26) entailed that man could discern mathematical patterns in creation. Mathematics was held to be true because of its supposed divine origin. The notion of a rational Creator was vigorously espoused by Kepler, Galileo, Newton, and many other early scientists. It was a major factor in motivating the scientific revolution.

The rise of materialism

Ironically, the very success of mathematical models in physics led to the demise of the classical Christian view. First, the clockwork universe of Newtonian physics no longer seemed to need a God to run it. Moreover, a world entirely explicable in terms of natural laws contradicted the supernatural events related in the Bible, thus undermining biblical authority. Thus, Christianity was gradually replaced with naturalism, which tried to explain everything in terms of purely natural processes. The natural sciences came to be seen by many as the only means of acquiring truth about the world. God was either denied outright or banished to insignificance.

Most naturalists are *materialists*, believing that everything—even consciousness and mind—is just a form of matter. Materialists assume that everything in the universe is ultimately explicable in terms of material properties and interactions. Materialism has a very long history, dating back to Democritus of Abdera (ca 460 – 370 BC). He held that the world consisted only of atoms, emptiness, and motion. Everything else was formed through random interactions between the atoms moving through infinite empty space. Asserting that the universe had existed since eternity, Democritus tried to banish both creator and designer.

The notion that all causes are *physical* causes leads to *empiricism*, which limits our knowledge to what we learn through our physical senses. Empiricism rules out all non-sensory experience, such as innate knowledge, intuition, extra-sensory perception, or divine revelation.

Materialists believe that mathematical objects exist only materially, in our brains.[3] Mathematical objects are believed to correspond to physical states of our brain and, as such, should ultimately be explicable by neuroscience in terms of biochemical laws. Stanislas Dehaene suggests that human brains come equipped at birth with an innate, wired-in ability for mathematics.[4] He postulates that, through evolution, the smallest integers (1, 2, 3 ...) became hard-wired into the human nervous system, along with a crude ability to add and subtract. A similar position is defended by George Lakoff and Rafael Nunez, who seek to explain mathematics as a system of metaphors that ultimately derive from neural processes.[5] Penelope Maddy conjectures that our nervous system contains higher-order assemblies that correspond to thoughts of particular sets.[6] She posits that our beliefs about sets and other mathematical entities come, not from Platonic ideal forms, but, rather, from certain *physical* events, such as the development of pathways in neural systems. Such evolutionary explanations seek to derive all our mathematical *thoughts* from purely *physical* connections between neurons.

Problems with materialist mathematics

(1) Can materialism account for mathematical ability?

Thus far, such proposals for simple arithmetic are entirely hypothetical: no actual mathematical mechanisms have as yet been found in the brain. Yet, even if the evolutionary mechanism of random mutation and natural selection could account for an innate ability for simple arithmetic, it is hard to see where more advanced mathematics comes from. An ability for simple arithmetic might be useful for survival. However, our capacity for advanced mathematics seems to be well in advance of mere survival skills. Paul Davies comments:

> One of the oddities of human intelligence is that its level of advancement seems like a case of overkill. While a modicum of intelligence does have a good survival value, it is far from clear how such qualities as the ability to do advanced mathematics... ever evolved by natural selection. These higher intellectual functions are a world away from survival 'in the jungle'.... Most biologists believe the... human brain has changed little over tens of thousands of years, which suggests that higher mental functions have lain largely dormant until recently. Yet if these functions were not explicitly manifested at the time they were selected, why were they selected? How can natural selection operate on a hidden ability? Attempts to explain this by supposing that, say, mathematical ability simply piggy-backs on a more obvious useful trait are unconvincing in my view.[7]

(2) Why should materialist mathematics be true?

If our mathematical ideas are just the result of the physics of neural connections, why should they be true? Such accounts of mathematics cannot distinguish true results from false ones. Nor can they yield any explanation for *correctness*, a basic issue in mathematics. Indeed, if all knowledge is based on neural connections, so is the idea that all knowledge is based on neural connections. Hence, if true, we have no basis for believing it to be true.

David Ray Griffin notes that, according to materialism, all causation is *efficient* causation, meaning that each event is completely caused by previous events.[8] Rational thought, however, is guided by goals and *norms*, such as the rules of logic. As such, rational activity reflects *final* causation—causation in terms of a norm or goal—which is quite different from efficient causation. Since materialism equates the mind with the brain, whose activities are presumed to be completely determined by the physical activities of its parts (e.g. brain neurons), materialism has no room for final causation.

This raises the question: If everything can be explained in terms of physics, where do logical norms enter into our thinking? Note that it *is* possible for a *physical* mechanism to do *logical* operations. In a computer, for example, there is an exact correlation between the flow of physical states of the computer and the corresponding flow of logical operations. Here the correspondence is specifically designed by an intelligent agent. However, if the origin of the brain is attributed to a purposeless process then we have no grounds for believing in a perfect correspondence between the brain's physical flow and the mind's logical thinking. Further, in the case of the computer, the output is meaningless unless it is *interpreted* by an intelligent observer. In a purely material brain, where is there room for an intelligent interpreter?

(3) Does materialism have room for truth?

Central to mathematics (and rationality) are the notions of truth and logic. The common sense *correspondence* view of truth is that a proposition or belief is true if and only if it corresponds with what is actually the case.

Knowing something about reality involves the capacity to *represent* some aspect of reality as a thought in our mind. Our beliefs are tentative representations of reality. Our beliefs are judged either true or false depending on how well they represent reality.

Truth and falsity are objective properties of our *representations*, not of the external world itself. In themselves, physical objects do not represent anything. They do not refer to anything beyond themselves. Of course, they can be *interpreted* by us as representing something other than themselves, but the actual representation is then our mental interpretation. Dallas Willard argues that no *physical* property or combination of properties can constitute a *representation* of anything.[9] Hence, truth cannot be reduced to a physical property. It follows that truth cannot be explained by materialism.

Closely related to truth is *logic*. Logical propositions are either true or false. Logical laws and relations connect the truth-values of different propositions. Since truth is not a physical property, it follows that neither is logic. Moreover, logical laws are quite different from laws of physical or psychological fact.

Logical laws are neither hypothetical nor inductive but, rather, necessary and universal. They remain valid, regardless of the state of the physical world. Hence, they cannot be proven from any physical laws or state of affairs. Logical laws, like truth, are abstractions. As such, they belong to the realm of ideas, not matter. J. P. Moreland contends that consistent materialistic naturalism must reject abstract objects of any kind (including sets, numbers, propositions, and properties), if we take these in the traditional sense of being non-physical.[10]

These considerations raise particular problems for a materialist view of mathematics. Most mathematicians believe that numbers, equations, perfect circles, and so on, exist in some ideal, abstract sense. Such non-physical objects must be rejected by consistent materialists. However, if ideal entities do not exist, this means that any propositions concerning them cannot be true in the sense of corresponding to anything. As Griffin points out, one is then forced either to reduce mathematics to a mere game with meaningless symbols or to think of mathematical objects as part of the physical world, which is clearly not the case.[11] Consequently, few mathematicians are materialists.

(4) Can we eliminate universals?

If materialism has no place for ideal entities, then it must deny also the existence of universal norms. This affects not just mathematics but rationality in general. Rationality concerns the rightness or wrongness of our reasoning. It assumes the existence of objective, rational ''oughts'' that prescribe how we are to reason. Given certain arguments and evidence, a rational person *ought* to accept the conclusions they entail. This implies the existence of objective laws of logic and rules of evidence.

John L. Mackie denied the truth-value of moral claims because he thought that objective moral values must then exist in some ideal world. How, he asked, could such non-physical norms affect our mind so that we could come to know them? Mackie was convinced that one would have to appeal to some occult faculty of intuition, which he rejected. Mackie's naturalism committed him to the belief that the world consists solely of the physical and psychological phenomena that are the objects of natural science. His facts were limited to the way things are and what we think or do. They did not include how we *ought* to think or act. Consequently, Mackie concluded that postulating objective moral values is incoherent, at least within a naturalist worldview.[12] Later, Mackie judged that even *subjective* moral properties are difficult to fit into a naturalist world. How can ''is'' ever give rise to ''ought?'' Mackie contended: ''Moral properties constitute so odd a cluster of properties and relations that they are most unlikely to have arisen in the ordinary course of events without an all-powerful god to create them.''[13] This conclusion led Mackie to reject all moral properties.

Charles Larmore notes that *moral* oughts are similar in nature to *rational* oughts. Both are ideal and abstract. The notion of moral truth is no more dubious than the idea of there being a truth or falsity to any claim that something ought to be believed.[14] Hence, Mackie's argumentation can just as well be applied to rational norms. Mackie must then conclude that objective rational norms, too, are inconsistent with naturalism. This destroys the very idea of rationality.

Larmore asserts that, whereas *natural* facts are found by observation and experiment, *normative* facts involve *reasons*, which are found by reflection. He writes:

> The inadequacy of naturalism is in the end its inability to account for normative truth in general. Thus, the minute we suppose it is true that we ought to believe something, we have broken with the naturalistic perspective. Acknowledging that there are indeed reasons for belief and action is enough to dispel the mystery.... By leaving no room for there being reasons for belief, naturalism contradicts itself. Or it does if it presents itself as the truth regarding what we ought to believe about the world...[15]

In sum, materialism has no place for non-physical, universal norms. There can be no absolute standards of right or wrong mathematics. Even if there were, empiricism denies that we can acquire knowledge of such norms. After all, we can only observe what *is*, not what *ought to be*. Hence, naturalism, must postulate that all norms—whether rational, mathematical, or moral—are purely human inventions. Truth and falsity, right and wrong, and good and evil are thus reduced to mere human opinion or convention.

Thus, for example, Michael Ruse argues that human rationality is determined solely by genetic traits developed via the evolutionary struggle for survival.[16] However, such a claim undermines any claim of Ruse that his thinking is rational. As Hilary Putnam notes, "if rationality were measured by survival value, then the proto-beliefs of the cockroach... would have a far higher claim to rationality than the sum total of human knowledge."[17] Putnam, who had earlier denied the existence of any ideal truths, concedes that the law of non-contradiction, at least, is an absolutely unrevisable ideal truth.

Constructivism

The rejection of theism, with the consequent concerns for the *soundness* of mathematics, had implications also for the actual *content* of mathematics. Classical mathematics was based on the concept of an Ideal Mathematician. It assumed the existence of an all-knowing, all-powerful, and infinite God. The operations and proofs allowed in classical mathematics were those that could in principle be performed by such a God.

Some naturalist mathematicians, considering mathematics to be no more than the free creation of the *human* mind, felt that the methods of mathematics should be adjusted accordingly. Only those mathematical concepts and proofs were to be considered valid that could be (mentally) constructed in a finite number of explicit steps. The "there exists" of classical mathematics was to be replaced by "we can construct.'' Accordingly, this came to be known as *constructive* mathematics. As the constructionist mathematician Errett Bishop notes:

> Classical mathematics concerns itself with operations that can be carried out by God . . . . You may think that I am making a joke . . . by bringing God into the discussion. This is not true. I am doing my best to develop a secure philosophical foundation . . . for current mathematical practice. The most solid foundation available at present seems to me to involve the consideration of a being with non-finite powers—call him God or whatever you will—in addition to the powers possessed by finite beings.[18]

Bishop himself rejected classical mathematics and urged a constructive approach to mathematics. He writes,

> Mathematics belongs to man, not to God. We are not interested in properties of the positive integers that have no descriptive meaning for finite man. When a man proves a positive integer to exist, he should show how to find it. If God has mathematics of his own that needs to be done, let him do it himself.[19]

Constructive mathematics entailed a new approach to both logic and proofs. Consider, first, the implications for logic. Classical mathematics is based on what is called *two-valued* logic. Any well-posed mathematical proposition is either true or false; there is no third option. Thus, for example, Goldbach's Conjecture is either true or false, even though we do not yet know which it is. This follows from the logical Law of Excluded Middle, which asserts that any proposition is either true or false. Any other possibility is excluded.

Constructionists, however, object to the Law of Excluded Middle. They insist that there *is* a third possibility: a proposition is neither true nor false until we can construct an actual, finite proof. This radical view of logic places severe restrictions on what constructionists accept as valid proofs. For example, constructionists object to *proofs by contradiction* since these rely on the Law of Excluded Middle.

Constructionism entails the rejection of many results of classical mathematics. Modern physics, however, relies heavily on advanced mathematical concepts such as, for example, ''infinite Hilbert spaces'' in quantum mechanics, the ''Hawking-Penrose singularity theorems'' in general relativity, and ''renormalization'' in quantum electrodynamics. These are beyond the range of current constructive mathematics. Indeed, Geoffrey Hellman contends that it is impossible to reformulate quantum theory without resorting to the Law of Excluded Middle.[20] Thus, if one is to believe in the truth of these theories in modern physics, then one must accept the truth also of the advanced classical mathematics that these theories presume. This, in turn, entails the truth of mathematical realism and the Ideal Mathematician.

The indispensability of mathematics for science

This brings us to a deeper problem. Mathematics is indispensable to physics, which is essential to materialism. Willard Quine, who is otherwise a materialist, has argued that the indispensability of mathematics to science gives us good grounds to believe in the objective existence of mathematical entities, such as sets and functions.[21] Since we ought to be ontologically committed to those entities that are indispensable to our best scientific theories, and since these include mathematical entities, he concludes that we should thus be ontologically committed to mathematical entities. Further, since belief in other theoretical entities is justified by empirical evidence that confirms the theory as a whole, that same evidence similarly justifies belief in mathematical entities. If a scientific theory is confirmed by empirical data, then the *whole* theory is confirmed, including whatever mathematics the theory uses. A recent defense of realism based on the indispensability argument is given by Mark Colyvan.[22]

Since science deals with real objects, it would seem that mathematics must also deal with real objects. This applies even more so for those embracing a realist view of scientific theories. How can scientific theories be true unless the underlying mathematics is also true?

Hartry Field, an anti-realist, considers Quine's indispensability argument to be the strongest argument for realism. To defeat it, Field has tried to prove that mathematics is not essential to physics.[23] He has had some success with Newtonian mechanics. Nevertheless, it seems very unlikely that mathematics can be removed from more sophisticated theories, such as general relativity and quantum mechanics.

The applicability of mathematics

If mathematics is merely a human invention, why do relatively simple mathematical theories yield such accurate representations of the physical world? Sophisticated theories, such as relativity or quantum mechanics, can be aptly summarized in just a few small mathematical equations and their logical implications. The amazing success of physics is largely due to its basic mathematical nature. This suggests that the physical world reflects the same mathematical structure that mathematicians explore. Eugene Wigner commented on the amazing applicability of complex analysis to quantum mechanics:

> It is difficult to avoid the impression that a miracle confronts us here, quite comparable to the . . . miracles of the existence of laws of nature and of the human mind's capacity to divine them.[24]

> The miracle of the appropriateness of the language of mathematics for the formulation of the laws of physics is a wonderful gift which we neither understand nor deserve.[25]

Mark Steiner notes that Wigner's "mystery" is open to various objections.[26] First, Wigner ignores the failures, those instances where appropriate mathematical descriptions could not be found. In addition, many mathematical concepts have not yet been shown to have any practical applications.

Nevertheless, Steiner believes that Wigner is on to something. He contends that the applicability of mathematics concerns not just a few isolated successes in physics. Rather, it pertains to the much broader applicability of mathematics as a global research strategy. Physicists, from Kepler and Galileo onwards, have been gripped by the conviction that mathematics is the ultimate language of the universe. Physicists probe nature with an eye for mathematical structures and analogies.

However, such a mathematical research strategy for making discoveries is essentially an *anthropocentric* (i.e. man-centered) strategy. It presumes that humans have a special place in nature. This is because mathematics relies on human standards such as simplicity, elegance, beauty, and convenience. Anthropocentrism is most blatant in those cases where even the *notation* of mathematics plays a major role in scientific discovery. Steiner gives various historical examples, such as Paul Dirac's discovery of positrons and the formalism of quantum mechanics.

The philosophical problem is not just the applicability of mathematics to our *descriptions* of physical reality but, even more, the major role of *human* mathematics in the *discovery* of new phenomena. Steiner concludes that our universe appears to be intellectually "user friendly" to humans. This presents naturalism with a perplexing problem. If we are mere accidents in a purposeless world of matter then we cannot expect the universe to reflect our standards of beauty and convenience. How, then, are naturalists to account for the fact that the universe's mathematical structure is just simple enough for *humans* to discern?

Steiner's examples of the amazing use of mathematics, in both scientific description and discovery, argue strongly against the notion that mathematics is merely a human invention.

In sum, the applicability of mathematics favors realism. Further evidence can be given for realism. This includes the *universality* of mathematics, the fact that mathematicians widely separated in space, time, and culture find the same mathematical theorems and ideas. The evidence includes also the strong sense of discovery mathematicians have when finding new theorems, mathematical intuition, and the fact that realism is the working philosophy of most mathematicians. In the last century, realism has been explicitly defended by a number of outstanding mathematicians, including Georg Cantor, Kurt Gödel, G. H. Hardy, and Roger Penrose. For example, Roger Penrose writes that "like Everest, the Mandelbrot set is just there" and "there is something absolute and 'God given' about mathematical truth."[27] Likewise, Hardy believed that "mathematical reality lies outside us . . . our function is to discover or observe it . . . the theorems which we prove, and which we describe grandiloquently as our 'creations,' are simply our notes of our observations."[28]

Problems with naturalist realism

Realism, however, does raise some difficult questions. First, there is the question of where and how such mathematical entities exist. Second, Colin Cheyne and Charles Pigden note that in realism mathematical objects are causally inert.[29]

How, then, they ask, can mathematics be responsible for the world's being the way it is? Further, if mathematical entities are inert, how do they influence our minds? Paul Benacerraf contends true beliefs may be considered genuine knowledge only if their truth is *causally* responsible for our belief.[30] Since mathematical entities are causally inert, they cannot give rise to our mathematical knowledge.

It is clear that realism requires an active agent. In the theistic worldview this poses no problem, for mathematical objects can be causally effective in the world, and in our minds, by virtue of being in the mind of God. God can always cause the required connections to be made. Those who reject theism, however, are faced with a daunting problem. Griffin comments:

> The implication of Benacerraf's insight . . . is that atheism renders unintelligible the idea that we can have knowledge of a Platonic realm of numbers. Several philosophers of mathematics, including Hersch himself, use Benacerraf's insight as the basis for rejecting a Platonic realm. As Quine points out, however, such a realm is presupposed by physics. Benacerraf's insight, plus Quine's observation, implies that atheism makes an adequate philosophy of mathematics impossible.[31]

For example, Reuben Hersch finds that ''Recent troubles in philosophy of mathematics are ultimately a consequence of the banishment of religion from science.''[32] He concedes that ''Platonism . . . was tenable with belief in a Divine Mind . . . The trouble with today's Platonism is that it gives up God, but wants to keep mathematics a thought in the mind of God.''[33] ''Once mysticism is left behind . . . Platonism is hard to maintain.''[34] Hersch argues that believing in eternal mathematical objects existing independently of human minds is possible only if one believes that God exists, which, he says, no one does anymore.

Similarly, Yehua Rav comments:

> Whereas the quarrel about universals and ontology had its meaning and significance within the context of medieval Christianity, it is an intellectual scandal that some philosophers of mathematics can still discuss whether whole numbers exist or not.[35]

> There are no preordained, predetermined mathematical 'truths' that just lie out . . . there. Evolutionary thinking teaches us otherwise.[36]

Once theism is dropped, it is difficult for realism to explain where objective mathematical truths exist and how we have access to them. Mathematical realism is plausible, it seems, only within a theistic worldview.

As R. G. Collingwood has noted, pure Platonism holds no hope for applied mathematics because it views the physical world as merely a rough copy of the ideal. Collingwood finds that, historically, the possibility of applied mathematics comes only with the Christian belief of a rational, omnipotent God who created the world according to a purposeful plan.[37] Similarly, C. F. von Weizsacker observes that precise mathematical structure in matter requires an omnipotent Creator rather than crass Platonism. He concludes, ''In this sense I called modern science a legacy . . . of Christianity.''[38]

Mathematics in a theistic worldview

How does mathematics fit within a theistic worldview? We note first that the biblical God has a logical aspect ("the spirit of truth" [John 15:26]), as well as a numerical aspect (the tri-une God of Father, Son and Holy Spirit). Since God is eternal, so are logic and number. God is also infinite, omnipotent, and omniscient; His knowledge encompasses all events, thoughts and possibilities, including all possible mathematical propositions. God's upholds all truths, including truths about mathematics. Hence, a mathematical entity need not be explicitly constructed in order to exist.

As the omnipotent ground of all being, God upholds everything, even all possibilities, establishing what is possible and what is necessary. Rather than necessary truths diminishing God's sovereignty, the omnipotence of God is most dramatically illustrated by the fact that God establishes and upholds even whatever is possible and whatever is necessary.

How do humans come to know eternal mathematical truths? The Bible tells us that man was made in the image of God (Gen.1:26–30; I Cor.11:7), with the ability to rule God's creation (Gen.1:28). This image includes rationality and creativity. The ability to do mathematics seems to be innate in human minds. This involves the capacity for abstract thought, as well as the ability to discern and symbolize. God has formed our minds so that they can readily handle abstract thought, symbolic representation, and logical manipulation. Alvin Plantinga comments:

> God has... created us with cognitive faculties designed to enable us to achieve true beliefs with respect to a wide variety of propositions—propositions about our immediate environment, about our own interior lives, about the thoughts and experiences of other persons, about our universe at large, about right and wrong, about the whole realm of abstracta—numbers, properties, propositions—... and about himself.[39]

Since God has created the physical world according to a rational plan, the world may be expected to exhibit mathematical structure. Since God has created man in God's image, humans may be expected to discern this mathematical structure. This accounts for the applicability of human mathematics to the physical world.

As we noted earlier, classical mathematics is based on the notion of an infinite, omniscient, and omnipotent God. Thus, theism provides a basis for classical mathematics. Georg Cantor (1845–1918), the founder of modern set theory, justified his belief in infinite sets by his belief in an infinite God.[40] He thought of sets in terms of what God could do with them. An infinite God would have no difficulty forming the power set of any given infinite set. Even today, almost every attempt to motivate the principles of combinatorial set theory relies on some notion of idealized manipulative capacities of the Omnipotent Mathematician. Constructionists, however, reject Cantor's transfinite cardinal numbers since these cannot be constructed by finite methods.

Alvin Plantinga notes that theists have a distinct advantage when it comes to explaining sets and their properties. The existence of sets depends upon a certain sort of intellectual activity—a collecting or ''thinking together.'' According to Plantinga,

> If the collecting or thinking together had to be done by human thinkers, or any finite thinkers, there wouldn't be nearly enough sets—not nearly as many as we think in fact there are. From a theistic point of view, the natural conclusion is that sets owe their existence to God's thinking things together . . . Christians, theists, ought to understand sets from a Christian and theistic point of view. What they believe as theists affords a resource for understanding sets not available to the non-theist . . .[41]

A detailed theistic justification of set theory has been developed by Christopher Menzel.[42] Ultimately, the consistency and certainty of mathematics can be grounded upon the multi-faceted nature of God Himself. Trust in God generates confidence in mathematics.

Conclusions

In summary, mathematics is not plausibly explained by materialism. A materialist view of origins does not account for the ability of the human mind to do advanced mathematics. If mathematics is generated by a brain operating purely by physical causes, then there is no reason why such mathematics should be true. Indeed, the very notions of truth and logic do not fit well with materialism, which has no place for universals norms.

Classical mathematics is based on the notion of an infinite, omnipotent, and omniscient God. The rejection of such a being leads to constructivism. However, constructivism fails to support the sophisticated mathematics needed for physics. The indispensability of mathematics for physics is a strong argument for realism. The universality of mathematics, as well as its applicability to the physical universe, are further evidence that mathematics cannot be reduced to a mere human invention.

Realism is faced with the problems of where to place ideal mathematical entities and how humans can access such an ideal realm. These problems are difficult to solve without theism. Theism posits that God upholds mathematical truths, which reside in the divine mind. Classical mathematics presumes the existence of an Ideal Mathematician—an all-powerful, all-knowing and infinite God. Theism holds that man is created in God's image and that God can communicate truth to us. Hence, humans may be expected to have access to mathematical knowledge. Since God is a rational Creator, the physical universe may be expected to have a physical structure. Finally, theism provides the resources to justify the soundness of mathematics.

Endnotes

1 Bertrand Russell, *A History of Western Philosophy* (New York: Simon & Schuster, 1945), 37.
2 Norman Geisler and Winfred Corduan, *Philosophy of Religion*, 2nd ed. (Grand Rapids, Mich.: Baker, 1988), 154.

3 Cf. Jean-Pierre Changeux and Alain Connes, *Conversations on Mind, Matter, and Mathematics* (Princeton, N.J.: Princeton University Press, 1995), 13.
4 Stanislas Dehaene, *The Number Sense: How the Mind Creates Mathematics* (Oxford: Oxford University Press, 1997).
5 George Lakoff and Rafael Nunez, *Where Mathematics Comes From* (New York: Basic Books, 2000).
6 Penelope Maddy, *Realism in Mathematics* (Oxford: Oxford University Press, 1993).
7 Paul Davies, *Are We Alone?* (New York: Basic Books, 1995), 85–86.
8 David R. Griffin, "Naturalism: Scientific and Religious," *Zygon*, vol. 37, no. 2 (2002): 372.
9 Dallas Willard, "Knowledge and Naturalism," in *Naturalism: A Critical Analysis*, eds. William L. Craig and James P. Moreland (London: Routledge, 2000), 39.
10 James P. Moreland, "Naturalism and the Ontological Status of Properties," in *Naturalism*, eds. Craig and Moreland, 67.
11 Griffin, "Naturalism: Scientific and Religious," 369.
12 John L. Mackie, *Ethics: Inventing Right and Wrong* (Harmondsworth: Penguin, 1979), 38–41.
13 John L. Mackie, *The Miracle of Theism* (Oxford: Clarendon Press, 1982), 115.
14 Charles Larmore, *The Morals of Modernity* (Cambridge: Cambridge University Press, 1996), 99.
15 Ibid., 115–116.
16 Michael Ruse, *Taking Darwin Seriously* (Amherst, N.Y.: Prometheus Books, 1998), 206.
17 Hilary Putnam, *Realism and Reason* (New York: Cambridge University Press, 1983), 232.
18 Errett Bishop, "Schizophrenia in Contemporary Mathematics," in *Errett Bishop: Reflections on Him and His Research*, ed. Murray Rosenblatt (Providence, R.I.: American Mathematical Society, 1985), 9.
19 Errett Bishop, *Foundations of Constructive Analysis* (New York: McGraw-Hill, 1967), 2.
20 Geoffrey Hellman, "Constructive Mathematics and Quantum Mechanics: Unbounded Operators and the Spectral Theorem," *Journal of Philosophical Logic*, 23 (1993): 221–228; idem, "Quantum Mechanical Unbounded Operators and Constructive Mathematics—A Rejoinder to Bridges," *Journal of Philosophical Logic*, 26 (1997): 121–127.
21 Willard V. O. Quine, *Theories and Things* (Cambridge, Mass.: Harvard University Press, 1982), 14–15.
22 Mark Colyvan, *The Indispensability of Mathematics* (New York: Oxford University Press, 2001).
23 Hartry Field, *Science Without Numbers* (Oxford: Blackwell, 1980).
24 Eugene Wigner, "The Unreasonable Effectiveness of Mathematics," *Communications on Pure and Applied Mathematics*, 13 (1960): 8.
25 Ibid., 14.
26 Mark Steiner, *The Applicability of Mathematics as a Philosophical Problem* (Cambridge, Mass.: Harvard University Press, 1998).
27 Roger Penrose, *The Emperor's New Mind* (Oxford: Oxford University Press, 1989), 95, 112.
28 G. H. Hardy, *A Mathematician's Apology* (Cambridge: Cambridge University Press, 1967), 123–124.
29 Colin Cheyen and Charles R. Pigden, "Pythagorean Powers or a Challenge to Platonism," *Australian Journal of Philosophy*, 74 (1996): 639–645.
30 Paul Benacerraf, "Mathematical Truth," in *Philosophy of Mathematics*, 2nd ed., eds. Paul Benacerraf and Hilary Putnam (Cambridge: Cambridge University Press, 1983), 412.
31 Griffin, "Naturalism: Scientific and Religious," 373.
32 Reuben Hersch, *What Is Mathematics, Really?* (Oxford: Oxford University Press, 1997), 122.
33 Ibid., 135.
34 Ibid., 42.
35 Yehuda Rav, "Philosophical problems in the light of Evolutionary Epistemology," in *Math Worlds*, ed. Sal Restivo (Albany, N.Y.: State University of New York Press, 1993), 81.

36 Ibid., 100.
37 R. G. Collingwood, *An Essay on Metaphysics* (Lanham, Md.: University Press of America, 1984), 250–257.
38 C. F. von Weizsacker, *The Relevance of Science* (New York: Harper & Row, 1964), 163.
39 Alvin Plantinga, *Warrant and Proper Function* (Oxford: Oxford University Press, 1993), 201.
40 See Joseph W. Dauben, *Georg Cantor: His Mathematics and Philosophy of the Infinite* (Princeton, N.J.: Princeton University Press, 1979), 229.
41 Alvin Plantinga, ''Prologue: Advice to Christian Philosophers,'' in *Christian Theism and the Problems of Philosophy*, ed. Michael D. Beaty (Notre Dame, Ind.: University of Notre Dame Press, 1990), 35.
42 Christopher Menzel, ''An Activist Model of the Metaphysics of Mathematics,'' in *A Sixth Conference On Mathematics From A Christian Perspective*, ed. Robert L. Brabenec (Wheaton, Ill.: Wheaton College Mathematics Department, 1987), 55–72; idem, ''Theism, Platonism, and the Metaphysics of Mathematics,'' in *Christian Theism and the Problems of Philosophy*, ed. Michael D. Beaty (Notre Dame, Ind.: University of Notre Dame Press, 1990).

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