Data Assimilation Methods for the Evolution of Glacier using Level Set Method

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Introduction

Ice is an incompressible and non-Newtonian viscous fluid with extremely low Reynolds number flow.

A level set method handles topological changes of glaciers and the evolution of the ice-air or ice-water interface.

For short-term ice dynamics prediction, an optimal fit between observations and model output is essential.

A level set method for modeling glacier together with data assimilation is developing for advancing and re-creating glaciers and initialization of ice-sheet models.

Ice-sheet Model

Ice Flow Equations

\[ \nabla \cdot \mathbf{v} = 0 \]

\[ \rho (\mathbf{v} \cdot \nabla \mathbf{v}) = -\nabla p + \nabla \cdot (\nu \nabla \mathbf{v}) + \rho g \]

Stokes Approximation

Ice is a slow moving fluid.

\( \rho (\mathbf{v} \cdot \nabla \mathbf{v}) \approx 0 \)

\( \nabla \cdot \mathbf{v} = 0 \)

\( \mathbf{v} = -\nabla p + \frac{\rho g}{\left( \frac{1}{n} \right)} \nabla \cdot \mathbf{v} + \frac{\rho g}{\left( \frac{1}{n} \right)} \nabla \cdot \mathbf{u} \)

\( \mathbf{u} \) is the strain rate tensor

\( \nu \) is the magnitude of \( \mathbf{u} \)

\( \nabla \cdot \mathbf{v} = \nabla \cdot \mathbf{v} \)

Shallow Ice Approximation (SIA)

\( H_t = M + \nabla \cdot (D \nabla h) \)

\( \Gamma = \frac{1}{\left( \frac{1}{n} \right)} \)

\( D = \nabla H + \nabla H^\top \)

\( n = 3 \) in Glen’s flow law

\( M \) is mass balance of ice

\( \rho \) is density of ice (1010 kg m\(^{-3}\))

Horizontal ice velocity,

\( w(x, z, t) = -\frac{\nabla h}{n+1} \left( H^{n+1} - (h-z)^{n+1} \right) [h_z]^{n+1} \)

and vertical velocity,

\( w(x, z, t) = -\frac{\partial}{\partial z} w(x, z, t) \)

Level Set Method

Experiment 1: P. Halfar found the similarity solution of the SIA in the case of flat bed (b(x) = 0) and no surface mass balance (M(x, t) = 0). Therefore the SIA equation becomes,

\[ H_t = \nabla \cdot \left( \nabla H + \frac{\rho g}{\left( \frac{1}{n} \right)} \nabla \cdot \mathbf{v} - \mathbf{v} \right) \]

Now, the ice-air interface is expressed by,

\[ \phi(x, t) = \frac{1}{2} |x - b(x, t)| \]

where \( \Omega \) represents the region inside ice and \( \partial \Omega \) represents the ice-air interface. The level set function \( \phi \) can be defined as a signed distance function,

\[ \phi(x, t) = \frac{1}{2} |x - b(x, t)|, \quad \text{if} \quad x \in \Omega, \]

\[ -\phi(x, t), \quad \text{if} \quad x \in \partial \Omega. \]

Results

Data Assimilation

Compute velocity \( \mathbf{v} \) from the Stokes equations

Compute surface evolution using level set method by imposing the velocity field \( \mathbf{v} \) and surface mass balance

Compute optimal surface evolution using observation data and data assimilation method and re-define level set function \( \phi(x, t) \) for prediction

Summary

A level set method is implemented for the solution of an ice-air interface subject to an imposed velocity field and surface mass balance

EKF is implemented for 1-D advection-diffusion equation with constant diffusivity

Future Work

Compute velocity \( \mathbf{v} \) from the Stokes equations

Compute surface evolution using level set method by imposing the velocity field \( \mathbf{v} \) and surface mass balance

Compute optimal surface evolution using observation data and data assimilation method and re-define level set function \( \phi(x, t) \) for prediction

References


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Figure 1: A marine ice sheet profile, image taken from Schoof [1].

Figure 2: (left) Grid points in the computational domain; (mid) Initial Level Set Function \( \phi(x, z, t = 0) \); (right) surface elevation \( z \) of the ice sheet.

Figure 3: Evolution of the interface subject to an imposed velocity field and surface mass balance.

Figure 4: EKF with 1-D advection-diffusion equation (diffusivity, \( \nu = 0.0 \)) at time, \( t = 2 \).