Coupling Glacial Hydrology into a High-Order Numerical Ice Model

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IASC-NAG Meeting, February 2009

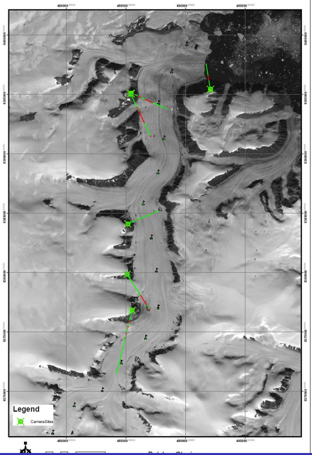
Aims

A new high-order flow-band model with coupled subglacial hydrology is used to explore the drainage of supraglacially-stored water through englacial fractures and assess the influence of this water on glacier dynamics

IPY Project: Belcher Glacier on Devon Island

Sharing similarities with many Greenland outlet glaciers, this is a large, fast-flowing, tidewater glacier

The Belcher Glacier System on the Devon Island Ice Cap



Model Overview

- **Flow-band**
  One horizontal dimension (in the direction of the ice flow), one vertical dimension, and a parameterization of the width across the flowline

- **Mass balance, evolving surface**

$$\frac{\partial h}{\partial t} = -\frac{1}{W}\frac{\partial(\bar{u}hW)}{\partial x} + M,$$

h	is the ice thickness
t	is time
W	represents the flowline width
$\bar{u}$	denotes the vertically averaged horizontal velocity
M	is the mass balance

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High-order stress components

- The model incoporates a multilayer longitudinal stress scheme following (Blatter, 1995) and (Pattyn, 2002)

- The vertical normal stress is assumed to be hydrostatic

$$\frac{\partial \sigma_{zz}}{\partial z} = \rho_i g$$

with the pressure (the sum of the normal stresses) departing from the hydrostatic pressure by the longitudinal deviatoric stress $P_i = \sigma'_{xx} + \sigma'_{yy} - \rho_i g(s - z)$

- Unlike the full stress solution the pressure doesn't include the integrated horizontal gradient in vertical shear stress

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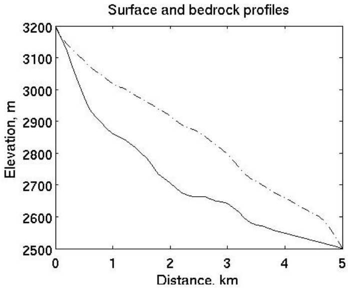
Model Comparison

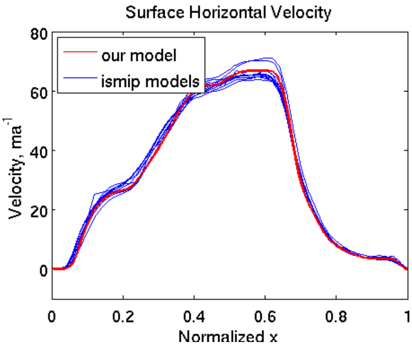
ISMIP-HOM Experiment E: Haut Glacier d'Arolla

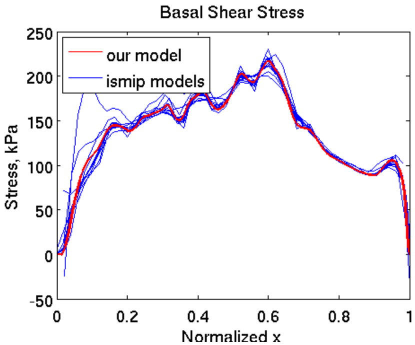


- test the velocity/stress solution of the non-linear force-balance equations
- use fixed geometry
- no-slip basal b.c.
- isothermal

ISMIP-HOM results taken from (Pattyn et al., 2008)

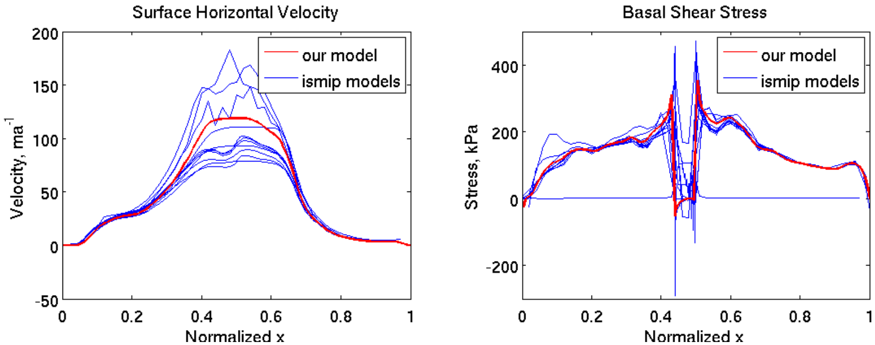
Without Basal Sliding





ISMIP-HOM results taken from (Pattyn et al., 2008)

With Local Basal Sliding



- **Thermomechanically coupled**
  - Solve an advective-diffusive heat equation
  - Temperature dependent flow-law parameter, as well as coupling from internal friction from deformational heating
  - Potential for coupling to the hydrological system

- **Calving rules**
  A suite of basic options for calving has been installed:
  - water-depth relation (Meier and Post, 1987)
  - flotation criterion (van der Veen, 1996)
  - crevasse formation (Benn et al., 2007)

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- **Sliding Law**
  - A Coulomb friction law is employed (Schoof, 2005)
- **Lateral Drag Parameterization**
  - Use the sliding law to determine sliding at the side walls and parameterize the effects of lateral drag
- **Coupled hydrology**
  - Vertical fracture propagation
  - Model subglacial drainage system
  - This will ultimately comprise ‘slow’/distributed and ‘fast’/channelized drainage networks

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Coulomb Sliding

$$\tau_b = C \left(\frac{u_b}{u_b + N^n \Lambda} \right)^{1/n} N, \qquad \Lambda = \frac{\lambda_{max} A}{m_{max}}$$

u_b	is the basal velocity
τ_b	is basal drag
N	the effective pressure
λ_{max}	a dimensional wavelength for the dominant bedrock bumps
m_{max}	a typical bed slope
A and n	Glen's flow law parameters
C	constant

This is a non-linear Robin-type boundary condition which cannot be solved independently but forms part of the solution to the ice-flow problem.

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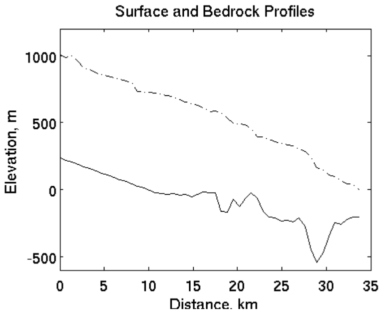
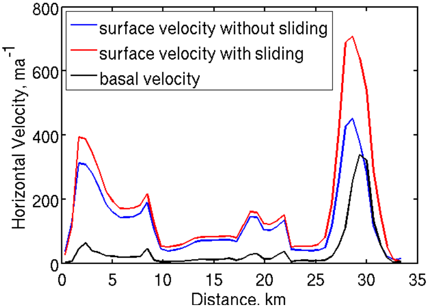
A profile along the Belcher flow line using the Dowdeswell et al. (2004) radar survey





Lateral Drag

We parameterize the lateral drag at the side walls as follows:

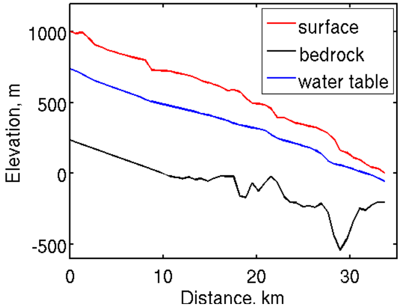
$$\sigma_{xy} \approx -\frac{\nu(u - u_L)}{W},$$

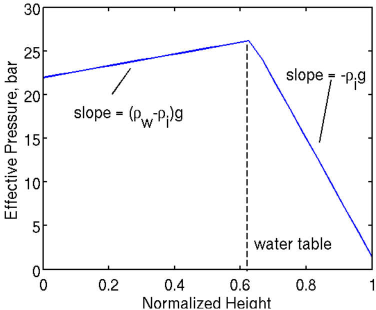
where u_L is the lateral sliding along the side walls and is computed by applying the Coulomb friction law

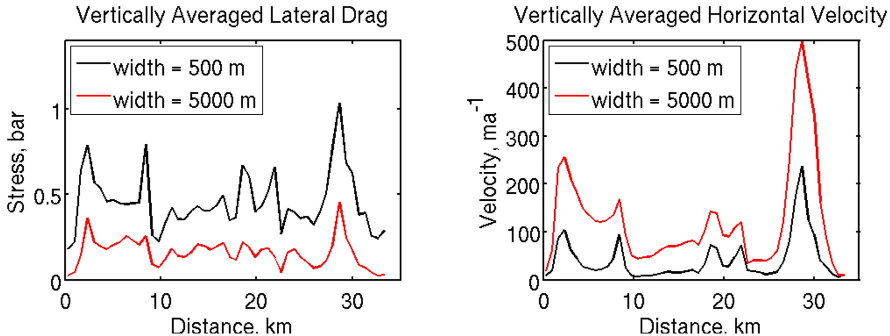
$$\sigma_{xy} = C\left(\frac{u_L m_{max}}{u_L m_{max} + N_L^n \lambda_{max} A)}\right)^{1/n} N_L,$$

here N_L is the vertical distribution of effective pressure along the side walls

Effects of width on Lateral Drag






Vertically Averaged Lateral Drag
width = 500 m
width = 5000 m
Stress, bar
0
0.5
1
0
10
20
30
Distance, km
Vertically Averaged Horizontal Velocity
width = 500 m
width = 5000 m
Velocity, ma^-1
0
100
200
300
400
500
0
10
20
30
Distance, km

Coupled Hydrology

Subglacial water drainage (Flowers and Clarke, 2002):
Conservation of mass in subglacial water sheet

$$\frac{\partial h^s}{\partial t} + \frac{\partial Q}{\partial x} = \frac{Q_G + u_b \tau_b}{\rho L} + M_b, \qquad Q = -\frac{K h^s}{\rho_w g} \frac{\partial \phi}{\partial x},$$

h^s	water sheet thickness
Q	water flux
$\phi = P_w + \rho_w g b$	fluid potential
$K = K(h^s)$	hydraulic conductivity
$P_w = P_i \left(\frac{h^s}{h^s_c}\right)^{7/2}$	water pressure
h^s_c	critical water sheet thickness

Vertical fracture progagation: linear elastic fracture mechanics (van der Veen, 2007)

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A Drainage Scenario

An experiment to mimic the drainage of a supraglacial lake in an effort to understand the coupling between hydrology and ice dynamics

1. Supraglacial drainage into crevasse englacial fracture propagation
2. Rapid drainage of meltwater pond into the subglacial drainage system
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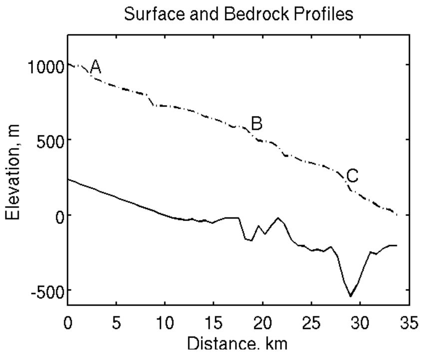
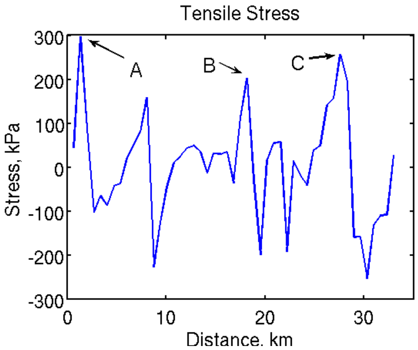
Englacial Fracture





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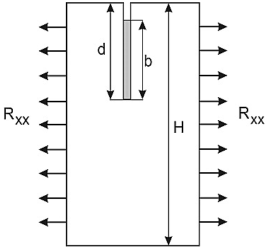
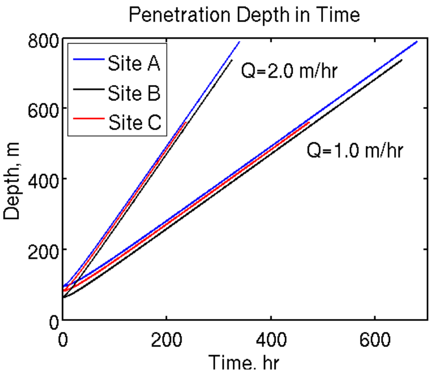


Reproduced from (van der Veen, 2007)

R_{xx}	far-field tensile stress
H	ice thickness
d	crevasse depth
$b = Qt$	water level
Q	water injection rate



Subglacial Drainage

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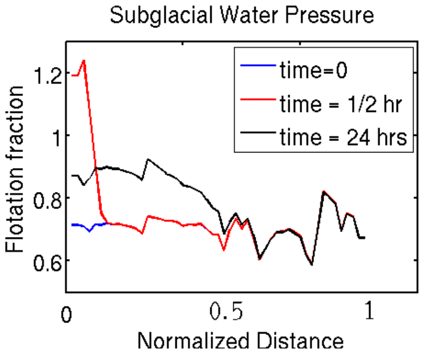
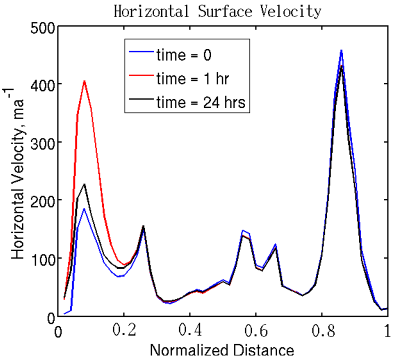
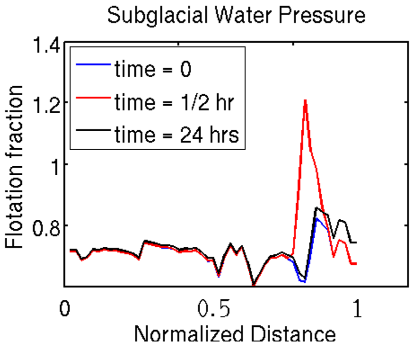
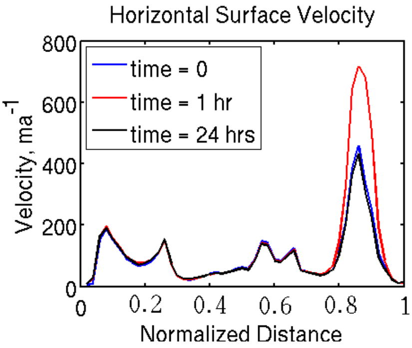




Subglacial Drainage

Site C





Coming Soon

- Extend coupled hydrology to include conduit equations - need a fast drainage system
- Fast and slow elements will be dynamically coupled to allow spatial and temporal changes in drainage morphology
- Simulations indicate that conduit systems develop rapidly in response to rapid water injection
- High water pressures result in vertical uplift caused by elastic flexure of the ice
- Model this uplift using an elastic beam equation

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Questions?