2013 03 18 $\frac{}{46}$ Math 124 Midterm #2 KEY

**I [9] Place the answers in the blanks to the right of the question:**

1. In sequences described by formulae like $\left\{\frac{n}{n^2-3}\right\}$ the domain of n is either $\mathbb{W}$ or $\mathbb{N}$

2. What do the terms of the sequence in #1 converge to? 0

3. Why doesn't the series $\sum_{i=1}^{\infty}(1)^{2i}$ converge? $(1)^{2i} \not\to 0$ The series is $1+1+1+1 \ldots \to \infty$

4. $\sum_{n=0}^{\infty}\cos \pi n$ is an alternating series ~~of decreasing terms~~, yet it does not converge. Why not?
The series is $1+-1+1+-1$ and the partial sums are $1,0,1,0,1,0\ldots$ which doesn't converge

5. What is the general form(ula) of a Taylor series? $\sum_{n=0}^{\infty}(-1)^n(x-a)^n$

6. $e^{i\pi} =$ $-1$ 7. $\sqrt[3]{8(\cos\frac{\pi}{2}+i\sin\frac{\pi}{2})} =$ $2(\cos\frac{\pi}{6}+i\sin\frac{\pi}{6})$ or $\sqrt{3}+i$

8. Simplify $(a+bi)(a-bi)$ $a^2+b^2$ 9. $i^{4n+1} =$ $i$

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**II [6] Find the limits of the sequences (or say they do not exist) SHOW WORK**

10. $\left\{\frac{n}{\ln n}\right\}$

consider $\lim_{x\to\infty}\frac{x}{\ln x}$ $\frac{\infty}{\infty}$

$\overset{LH}{=} \lim_{x\to\infty}\frac{1}{\frac{1}{x}}$

$= \lim_{x\to\infty} x$

$= \infty$

(does not exist)

11. $\left\{\frac{\cos 2n\pi}{n}\right\} = \left\{\frac{1}{n}\right\}$

and $\lim_{n\to\infty}\frac{1}{n} = 0$

12. $a_n = \sqrt{\frac{n^2+n+1}{n^3}} \times \frac{\sqrt{\frac{1}{n^3}}}{\sqrt{\frac{1}{n^3}}}$

$= \sqrt{\frac{\frac{1}{n}+\frac{1}{n^2}+\frac{1}{n^3}}{1}}$

$= \sqrt{\frac{0}{1}}$

$= 0$

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**III [9] Test for convergence. SHOW WORK**

13. $\sum_{n=2}^{\infty}\frac{n}{n^3+1}$ 779 # 11

$= \sum_{n=2}^{\infty}\frac{1}{n^2+\frac{1}{n}}$

and these terms are $< \frac{1}{n^2}$ (a converging p-series) so it converges

14. $\sum_{n=1}^{\infty}\frac{n^3}{5^n}$ 779 # 13

Ratio test

$\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| =$

$\lim_{n\to\infty}\left(\frac{(n+1)^3}{5^{n+1}} \times \frac{5^n}{n^3}\right)$

$= \lim_{n\to\infty}\left(\frac{n+1}{n}\right)^3 \times \frac{1}{5}$

$= 1 \times \frac{1}{5}$

$= \frac{1}{5} < 1$

converges

15. $\sum_{n=1}^{\infty}\left(1+\frac{3}{n}\right)^{-4n}$ see 779 # 7

Consider the terms $\frac{1}{\left(1+\frac{3}{n}\right)^{4n}}$

or $\frac{1}{\left[\left(1+\frac{3}{n}\right)^n\right]^4}$

Now as $n\to\infty$ we get $\left(\frac{1}{e^3}\right)^4 = \frac{1}{e^{12}} \neq 0$

so since the $a_n \not\to 0$ the series diverges.

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**IV [6] Determine whether the following converge absolutely, conditionally, or not at all**

16. $\sum_{n=1}^{\infty}(^-1)^n \frac{n}{2n^2+1}$

since the terms are positive, decreasing and $\lim_{n\to\infty} \frac{n}{2n^2+1} = \frac{\frac{1}{n}}{2+\frac{1}{n^2}} = \frac{0}{2} = 0$ the ACT tells us this thing converges. However If, we consider the absolute series, and do a limit comparison test with $\{\frac{1}{n}\}$ we have $\lim_{n\to\infty} \frac{\frac{n}{2n^2+1}}{\frac{1}{n}} = \lim \frac{n^2}{2n^2+1} = \frac{1}{2} \neq 0$ so since $\sum \frac{1}{n}$ diverges, this one does too.

Conditional

17. $\sum_{k=1}^{\infty}(^-1)^k \frac{\sqrt{2k}}{k+1} = \sqrt{2}\sum_{k=}(-1)^k \frac{k^{\frac{1}{2}}}{k+1}$

Considering $\frac{k^{\frac{1}{2}} \times \frac{1}{k}}{k+1 \times \frac{1}{k}} = \frac{k^{-\frac{1}{2}}}{1+\frac{1}{k}}$ the limit here is $\frac{0}{1} = 0$ so it converges (terms are positive, decreasing to zero)

Now note that $\frac{k^{\frac{1}{2}}}{k+1} = \frac{1}{k^{\frac{1}{2}}+1} \to \frac{1}{k^{\frac{1}{2}}}$ which is a p-series with $p<1$ so it diverges.

Hence conditional

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**V [4]Do one of these two**

18. What are the interval and radius of convergence?

$\sum_{n=1}^{\infty}(^-2)^n\left(\frac{x^n}{n!}\right) = 2\sum(-1)^n \frac{x^n}{n!}$

$\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty}\left|\frac{(-1)^{n+1}x^{n+1}}{(n+1)!} \times \frac{n!}{(-1)^n x^n}\right|$

$= \lim_{n\to\infty}\left|(-1)x \times \frac{1}{n+1}\right| = 0$

so this converges $\forall x$, ie $\underline{R=\infty}$

$\underline{I = (-\infty,\infty)}$

19. What does the series converge to:

$1-e+\frac{e^2}{2!}-\frac{e^3}{3!}+\frac{e^4}{4!}-\frac{e^5}{5!}....$

This is the series for $e^x$ but with $x$ replaced by $-e$, hence it converges to $e^{-e}$ or $\frac{1}{e^e}$ (.0660)

Yikes that was easy.

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**VI [8] Do TWO of these, clearly indicating which ones you want marked.**

20. Actually construct the Maclaurin series for $\sin x$. 21. Find $(2-i)^{12}$

22. Show by integrating the series for sin(x) that $\int \sin x dx = -\cos x + C$

23. Find a series for $f(x) = \frac{1}{1+x}$ 24. Find a series for $f(x) = x^2e^x$ (these by any means)

25. Find the cube roots of $1 + i$ You may leave the answers in polar form.

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(20) See notes or text

(21) $r = \sqrt{2^2+1^2} = \sqrt{5}$

$\theta = \tan^{-1}\frac{-1}{2} \doteq -.4636$

so $(2-i)^{12} = (\sqrt{5})^{12}(\cos 12\theta + i\sin 12\theta)$

$= 5^6(.752 + .6589i)$

$\underline{11753 + 10296i}$

(22) $\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + ---$ whose integral is $\frac{x^2}{2} - \frac{x^4}{4!} + \ldots + C$

$= -\left(1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \ldots\right) + C_1$

and since $\cos(0) = 1$ $C_1 = 0$

(23) $\frac{1}{1+x} = \frac{1}{1-^-x}$ $a_1 = 1$, $r = -1$ so $1 - x + x^2 - x^3 \ldots = \underline{\sum_{n=0}^{\infty}(-1)^n x^n}$

(24) Since $e^x = \sum \frac{x^n}{n!}$ $x^2e^x = \underline{\sum \frac{x^{n+2}}{n!}}$

(25) $1+i$ : $\theta = \frac{\pi}{4}$ $r = \sqrt{2}$ so $z = \sqrt{2}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)$

whence the cube roots are $(2^{\frac{1}{2}})^{\frac{1}{3}}\left(\cos\frac{1}{3}\frac{\pi}{4} + i\sin\frac{1}{3}\frac{\pi}{4}\right) = 2^{\frac{1}{6}}\left(\cos\frac{\pi}{12} + i\sin\frac{\pi}{12}\right)$

$(2^{\frac{1}{2}})^{\frac{1}{3}}\left(\cos\frac{1}{3}\frac{9\pi}{4} + i\sin\frac{9\pi}{4}\right) = 2^{\frac{1}{6}}\left(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right)$

$(2^{\frac{1}{2}})^{\frac{1}{3}}\left(\cos\frac{1}{3}\left(\frac{17\pi}{4}\right) + i\sin\frac{1}{3}\left(\frac{17\pi}{4}\right)\right) = 2^{\frac{1}{6}}\left(\cos\frac{17\pi}{12} + i\sin\frac{17\pi}{12}\right)$

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