

I [10] Place the answers in the blanks to the right of the question:

1. If $f(x)$ agrees with a_n for all n beyond a certain point, what can you conclude about convergence?

2. What is a minimal necessary (but not sufficient) condition for a series to converge _____

3. Why doesn't the series $\sum_{i=1}^{\infty} (-2)^i$ converge? _____

4. $\sum_{n=1}^{\infty} (-1)^n (4 - \frac{2}{n})$ is an alternating series of decreasing terms, yet it does not converge. Why not?

5. What is the difference between a Taylor series and a Maclaurin series?

6. $e^{i\theta} =$ _____ 7. $\sqrt{4(\cos \frac{\pi}{3} + i \sin \frac{\pi}{3})} =$ _____ 8. Simplify $\frac{1}{i}$ _____

9. Simplify $(a+bi)\overline{(a+bi)}$ _____ 10. $i^{4n} =$ _____

II [6] Find the limits of the sequences (or say they do not exist) SHOW WORK

11. $\left\{ \frac{\ln n}{n} \right\}$

12. $\left\{ \frac{3 \cos 4n\pi}{2n} \right\}$

13. $a_n = \sqrt{\frac{n^2 + n + 1}{n^2}}$

III [9] Test for convergence. SHOW WORK

14. $\sum_{n=2}^{\infty} \frac{1}{n \ln n}$

15. $\sum_{n=1}^{\infty} \frac{n^5}{e^n}$

16. $\sum_{k=1}^{\infty} \frac{(k+1)!}{k^k}$

IV [6] Determine whether the following converge absolutely, conditionally, or not at all

17. $\sum_{n=1}^{\infty} (-1)^n \frac{n}{2n^2 + 1}$

18. $\sum_{k=1}^{\infty} (-1)^k \frac{3^{k+1}}{k}$

V [4] Do one of these two

19. What are the interval and radius of convergence?

$$\sum_{n=1}^{\infty} (-2)^n \left(\frac{x^n}{n!} \right)$$

20. What does the series converge to:

$$1 - \frac{e^2}{2!} + \frac{e^4}{4!} - \frac{e^6}{6!} \dots$$

VI [8] Do TWO of these, clearly indicating which ones you want marked.21. Actually construct the Maclaurin series for $\cos x$.22. Find $(1-i)^{20}$ 23. Show by integrating the series for $\sin(x)$ that $\int \cos x dx = \sin x + C$ 24. Find a series for $f(x) = \frac{1}{1-x^2}$ 25. Find a series for $f(x) = x^2 \cos x$ (these by any means)26. Find the cube roots of $1 - i$. You may leave the answers in polar form.**VII [4] Do ONE of these, clearly indicating which one you want marked.**267 Write out the first four terms of the Taylor series for $f(x) = \sqrt{x}$ for $a = 5$.28. How many terms of the expansion of $\sin(x)$ are needed to find $\sin\left(\frac{\pi}{6}\right)$ within .0001?

29. A ball is dropped from a height of 100m and on each bounce it reaches one third its previous height. What is the total vertical distance it travels?

30. Find the actual sum of the convergent series $\sum_{n=1}^{\infty} 4 \left(\frac{3^{n+1}}{7^n} \right)$ 31. Estimate $(1.05)^{\frac{3}{2}}$ correct to .001 by a binomial expansion.32. Use a geometric series to rewrite the repeating decimal $0.\overline{23}$ as a common fraction.**VIII [+3] Bonus (not marked unless you have 75% above)**32. Simplify $\cosh \pi + \sinh \pi$