2013 02 08 p /50 Math 124 Midterm #1 ____________

I [6] Place the answers in the blanks to the right of the question:

1. $\int_1^\infty \frac{1}{x^5}dx$ converges because $\frac{1}{x^5} < \frac{1}{x^2}$ which converges

2. $\int \sec^2 x\,dx$ $\tan x + C$

3. $\frac{d}{dx}\coth(x)$ $-\operatorname{csch}^2 x$

4. $\int \frac{\pi e}{x-7}dx$ $\pi e \ln|x-7| + C$

5. $\int(\sin x\cos^5 x)dx$ $-u^5 du$ $-\frac{1}{6}\cos^6 x + C$

6. $\int \frac{d}{dx}\sin^{-1}(x)$ $\sin^{-1} x + C$

II [6] Show a step or two, even if you think you can do it within your cranium

7. $\int \tan^3 y\sec^2 y\,dy$ $u^3\,du$

$= \frac{1}{4}u^4 + C$

$= \frac{1}{4}\tan^4 y + C$

or doing it another way

$\frac{1}{4}(\sec^4 y - 2\sec^2 y) + C$

8. $\int \frac{2x\,dx}{\sqrt{3x^2-5}}$ $u = 3x^2-5$, $du = 6x\,dx$, $2x\,dx = \frac{1}{3}du$

$= \frac{1}{3}\int u^{-\frac{1}{2}}du$

$= \frac{1}{3}\times 2u^{\frac{1}{2}} + C$

$= \frac{2}{3}\sqrt{3x^2-5} + C$

9. $\int(6we^{7w})dw$ $u = 6w$, $dv = e^{7w}dw$, $du = 6dw$, $v = \frac{1}{7}e^{7w}$

$= \frac{6}{7}we^{7w} - \frac{6}{7}\int e^{7w}dw$

$= \frac{6}{7}we^{7w} - \frac{6}{49}e^{7w} + C$

or $\frac{6}{7}e^{7w}\left(w - \frac{1}{7}\right) + C$

III [24] Do SIX of these, clearly indicating which ones you want marked

10. $\int(\sin x - \sin^3 x)dx$

$= \int \sin x(1-\sin^2 x)dx$

$= \int \sin x\cos^2 x\,dx$

$= -\int u^2 du$

$= -\frac{1}{3}u^3 + C$

$= -\frac{1}{3}\cos^3 x + C$

11. $\int \frac{\ln x\,dx}{x}$ $u = \ln x$, $du = \frac{1}{x}dx$

$= \int u\,du$

$= \frac{u^2}{2} + C$

$= \frac{1}{2}\ln^2 x + C$

12. $\int 5t^2\cosh t^3\,dt$ $u = t^3$, $du = 3t^2$, $\frac{5}{3}du = 5t^2dt$

$= \frac{5}{3}\int \cosh u\,du$

$= \frac{5}{3}\sinh u + C$

$= \frac{5}{3}\sinh t^3 + C$

13. $\int \frac{y+1}{y^2-y-6}dy$

$= \int \frac{y+1}{(y-3)(y+2)}dy$ $\frac{A}{y-3} + \frac{B}{y+2}$

so $A(y+2) + B(y-3) = y+1$

$A + B = 1$

$2A - 3B = 1$

$2A + 2B = 2$

$-5B = -1$

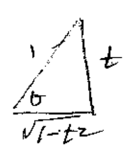
$B = \frac{1}{5}$

$A = \frac{4}{5}$

$= \frac{4}{5}\int \frac{1}{y-3}dy + \frac{1}{5}\int \frac{1}{y+2}dy$

$= \frac{4}{5}\ln|y-3| + \frac{1}{5}\ln|y+2| + C$

14. $\int \frac{\sqrt{1-t^2}}{t}dt$



$\sin\theta = t$

$\cos\theta = dt$

$\sqrt{1-t^2} = \cos\theta$

$= \int \frac{\cos^2\theta}{\sin\theta}d\theta$

$= \int \frac{1-\sin^2\theta}{\sin\theta}d\theta$

$= \int \frac{1}{\sin\theta} - \sin\theta\,d\theta$

$= \int(\csc\theta - \sin\theta)d\theta$

$= \ln|\csc\theta - \cot\theta| + \cos\theta + C$

$= \ln\left|\frac{1}{t} - \frac{\sqrt{1-t^2}}{t}\right| + \sqrt{1-t^2} + C$

15. $\int_{-5}^{5}(\cos^2 x - x^3)dx$

$= \int_{-5}^{5}\cos^2 x\,dx - \int_{-5}^{5}x^3 dx$

$= 2\int_0^5 \cos^2 x - 0$ 2^{nd} is (odd function)

$= 2\times\frac{1}{2}\int(1+\cos 2x)dx$

$= \left[x + \frac{1}{2}\sin 2x\right]_0^5$

$= 5 + \frac{1}{2}\sin 10 \doteq 4.728$

16. $\int_0^{\frac{\pi}{2}}\sqrt{1-\cos x}\,dx$ $\frac{\sqrt{1+\cos x}}{\sqrt{1+\cos x}}$

$= \int_0^{\frac{\pi}{2}}\frac{\sqrt{1-\cos^2 x}}{\sqrt{1+\cos x}}dx$

$= \int_0^{\frac{\pi}{2}}\frac{\sin x}{\sqrt{1+\cos x}}dx$ $u = 1+\cos x$, $du = -\sin x\,dx$ $u(0) = 2$, $u(\frac{\pi}{2}) = 1$

$= -\int_2^1 u^{-\frac{1}{2}}du$

$= -2u^{\frac{1}{2}}\Big]_2^1$

$= -2 + 2\sqrt{2} \doteq .414$

17. $\int_0^2 e^z\cos(e^z)dz$ $u = e^z$, $du = e^z dz$, $u(0) = 1$, $u(2) = e^2$

$= \int_1^{e^2}\cos u\,du$

$= \sin u\Big]_1^{e^2}$

$= \sin e^2 - \sin 1$

/36

u^3 — $e^{\frac{u}{3}}du$
$3u^2$ — $3e^{\frac{u}{3}}$
$6u$ — $9e^{\frac{u}{3}}$
6 — $27e^{\frac{u}{3}}$
0 — $81e^{\frac{u}{3}}$

IV [6] Do TWO of these, clearly indicating which ones you want marked.
For the first two, it is sufficient to say whether the integral converges or not (and why).

18. $\int_2^\infty 3e^{-3x}\,dx$

$= \lim_{t\to\infty}\int_2^t 3e^{-3x}dx$

$= \lim_{t\to\infty} -e^{-3x}\Big]_2^t$

$= \lim_{t\to\infty} -\left(\frac{1}{e^{3t}} - \frac{1}{e^6}\right)$

$= \frac{1}{e^6}$ converges. $\doteq .0025$

19. $\int_0^7 \frac{1}{\sqrt{x-7}}dx$

Without doing any work at all, I observe that I is undefined for $x<7$ hence it cannot converge.

20. $\int_0^1 u^3 e^{\frac{u}{3}}\,du$

$= 3u^3e^{\frac{u}{3}} - 27u^2e^{\frac{u}{3}} + 162ue^{\frac{u}{3}} - 486e^{\frac{u}{3}}\Big]_0^1$

$= \left(3e^{\frac13} - 27e^{\frac13} + 162e^{\frac13} - 486e^{\frac13}\right) - (-486)$

$= -348e^{\frac13} + 486 \doteq .327$

V [8] Do TWO of the questions in this section

21. Using n = 4 and Simpson's rule, find the approximate definite integral $\int_0^2 2^x\,dx$ $\quad \frac{b-a}{n} = \frac12$

$I \doteq \frac13\left[f(0) + 4f(\tfrac12) + 2f(1) + 4f(\tfrac32) + f(2)\right]\frac12 = \frac16(2^0 + 4\sqrt2 + 4 + 4\times2\sqrt2 + 4) = \frac16(9 + 12\sqrt2)$ or $\frac32 + 2\sqrt2 \cong 4.32$

22. Likewise using n = 5 and the midpoint rule, evaluate $\int_0^5 (3x^3 + 2x)\,dx$ $\quad \Delta x = 1$

$I \doteq \left[f(\tfrac12) + f(\tfrac32) + f(\tfrac52) + f(\tfrac72) + f(\tfrac92)\right]1 = \left[(\tfrac38+1) + (\tfrac{3\times27}{8}+3) + (\tfrac{3\times125}{8}+5) + (\tfrac{3\times343}{8}+7) + (\tfrac{3\times729}{8}+9)\right] = 459\frac38 + 25$
$= 484\frac38$

23. Find the centroid of $y = \frac12\sin 2x$ on the interval $0 \le x \le \frac{\pi}{4}$

24. What is the consumer surplus for a demand curve $p = 500 - .3x - .0005x^2$ when the ~~sales level~~ is 500?
$p(500) = 500 - 150 - 125 = 225$

25. The area bounded by $y^4 = x^3$, the y-axis and and x = 2 is rotated around the x-axis. Find the surface area of the resulting solid.

26. Find the ~~centroid of the area between $y = \sqrt{x}$ and~~ length of the curve $y = x^2$ on the interval $0 \le x \le 1$

27. $\int \frac{2x+5}{x^2+6x+8}dx$

$u = x \quad dv = \sin 2x\,dx$
$du = dx \quad v = -\frac12\cos 2x$

(23) $A = \int_0^{\frac{\pi}{4}} \frac12\sin 2x\,dx = -\frac14\cos 2x\Big]_0^{\frac{\pi}{4}} = \frac14$ $\quad \bar{x} = \frac1A\int_0^{\frac{\pi}{4}} x\,\frac12\sin 2x\,dx = 2\int_0^{\frac{\pi}{4}} x\sin 2x\,dx = 2\left[-\frac12 x\cos 2x - -\frac12\int\cos 2x\,dx\right]_0^{\frac{\pi}{4}}$

$= \left(-x\cos 2x + \frac12\sin 2x\right]_0^{\frac{\pi}{4}} = \left(0 + \frac12\right) - \left(-\frac{\pi}{4} + 0\right) = \frac{\pi}{4} + \frac12$ $\quad \bar{y} = \frac{1}{2A}\int_0^{\frac{\pi}{4}}\left(\frac12\sin 2x\right)^2dx = \frac12\int_0^{\frac{\pi}{4}}\frac{(1-\cos 4x)}{2}dx$ $\quad \cos 2\theta = 1 - \sin^2\theta$, $\sin^2\theta = \frac{1-\cos 2\theta}{2}$

$= \left[\frac14\left(x - \frac14\sin 4x\right)\right]_0^{\frac{\pi}{4}} = \frac14\left(\left(\frac{\pi}{4} + 0\right) - 0\right) = \frac{\pi}{16}$ $\quad$ Ans $\left(\frac{\pi}{4} + \frac12, \frac{\pi}{16}\right)$

(24) $CS = \int_0^{500}[p(x) - p]\,dx = \int_0^{500}[(500 - .3x - .005x^2) - 225]\,dx = \int_0^{500}(275 - .3x - .0005x^2)\,dx = \left(275x - \frac{.3}{2}x^2 - \frac{.0005}{3}x^3\right]_0^{500}$

$= 275(500) - \frac{.3(500^2)}{2} - \frac{.0005(500^3)}{3} = 137500 - 37500 - 20833.33 = \$79166.67$

(25) $y = x^3 \quad y' = 3x^2 \quad 1 + (y')^2 = 1 + 9x^4$ so $A = 2\pi\int_0^1 x^3\sqrt{1+9x^4}\,dx$ $\quad u = 1 + 9x^4$, $du = 36x^3dx$ so $x^3dx = \frac{1}{36}du$, $u(0) = 1$, $u(1) = 10$

$= 2\pi\int_1^{10}\frac{1}{36}\sqrt{u}\,du = \frac{\pi}{18}\cdot\frac23 u^{\frac32}\Big]_1^{10} = \frac{\pi}{27}\left(10^{\frac32} - 1\right) \doteq 3.56$ units^2

(26) $y = x^2$, $y' = 2x$ so $1 + (y')^2 = 1 + 4x^2$ whence $L = \int_0^1\sqrt{1+4x^2}\,dx$ $\quad \tan\theta = 2x$, $\theta = \tan^{-1}2x$, $\sqrt{1+4x^2} = \sec\theta$, $dx = \frac12\sec^2\theta$

$= \frac12\int_{x=0}\sec^3\theta\,d\theta = \frac12\left[\frac12\sec\theta\tan\theta + \frac12\ln|\sec\theta + \tan\theta|\right]_{x=0}^{1}$ (notes) $= \frac14\left(\sqrt5 \cdot 2 + \ln|\sqrt5 + 2| - (0 + \ln\sqrt5)\right)$
$= \frac14\sqrt5 + \frac12 + \ln|\sqrt5 + 2| - \ln\sqrt5$

(27) $\frac{2x+5}{(x+4)(x+2)} = \frac{a}{x+4} + \frac{b}{x+2}$ so $a(x+2) + b(x+4) = 2x + 5 \Rightarrow$ ① $a + b = 2$ ② $2a + 4b = 5$ $\quad$ $2\times$① $2a + 2b = 4$, $2b = 1$ $\therefore b = \frac12$, $a = \frac32$

$I = \frac32\int\frac{1}{x+4}dx + \frac12\int\frac{1}{x+2}dx = \frac32\ln|x+4| + \frac12\ln|x+2| + C$

VI Bonus (you should not waste time on these unless you are sure you have 75%+ above)

28. **[+2]** $\int\frac{\sin 2x - 5\sin x}{\cos^2 x - \cos x - 2}dx$

$= \int\frac{2\sin x\cos x - 5\sin x}{(\cos x - 2)(\cos x + 1)}dx$

$= \int\frac{2\cos x - 5}{(\cos x - 2)(\cos x + 1)}\sin x\,dx$ $\quad u = \cos x$, $du = \sin x\,dx$

$= \int\frac{2u - 5}{(u-2)(u+1)}du$ $\quad A(u+1) + B(u-2) = 2u - 5$; $A + B = 2$; $A - 2B = -5$; $3B = -3$; $B = -1$; $A = 3$

$= \int\frac{3}{u-2} + \frac{-1}{u+1}\,du$

$= 3\ln|\cos x - 2| - \ln|\cos x + 1| + C$

29. **[+2]** Get error bounds for *both* #21 and #22.

(21) $E_S \le \frac{k(b-a)^5}{180n^4}$ and $\left[2^x\right]^{iv} = \ln^4 2\cdot 2^x$ which on this interval is $\le \ln^4 2\cdot 2^2 < 1$

so $E_S \le \frac{1(2-0)^5}{180\cdot 4^4} \cong .0014$ (lower if k = exact)

(22) $E_M \le \frac{k(b-a)^3}{24n^2}$ and $f' = 9x^2 + 2$, $f'' = 18x$ which on $[0,5]$ is ≤ 90

So $E_S \le \frac{90(5-0)^3}{24\times 5^2} = \frac{90(5)}{24} = 18.75$

14